

Mahjong and mathematics

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Objectives of the presentation/expected learning outcomes

- Describe some background about the mahjong game.
One may see how cultural exchange happened.
- Study some of its mathematical aspects.
Learn the connection between mathematics and other activities.
- Describe some related research opportunities.
See how some research problems arise.

The following are extracted from the Wikipedia article on Mahjong.

- **Mahjong** (pinyin: **ma jiang**) is a game that originated in China.
- It is usually played by 4 persons (with 3-person or 2-person variation).
- Mahjong is a game of **skill**, **strategy** and **calculation** and involves a certain degree of **chance**.
- In Asia, mahjong is also popularly played as a gambling game though it may just as easily be played recreationally.

The origin

- One of the myths of the origin of mahjong suggests that **Confucius** (500 B.C.)
- Historians believed that the game was based on a Chinese card game called **Ma Diao**, in the early Ming dynasty.
- There is still some debate about who created the game.

One theory is that Chinese army officers serving during the Taiping Rebellion created the game to pass the time.

Another theory is that a nobleman living in the Shanghai area created the game between 1870 and 1875.

Others believe that two brothers from Ningpo - created mahjong around 1850, from the earlier game of Ma Diao.

Political and policy changes

- The game was banned by the **Communist Government** in 1949 because gambling was forbidden.
- After the **Cultural Revolution**, the game was revived, without gambling elements.
- The prohibition was revoked in 1985.
- Today, it is a favorite pastime in China and other Chinese-speaking communities.

Mahjong in Other Countries

- In 1895, Stewart Culin, an American anthropologist, wrote a paper in which mahjong was mentioned.
- This is the first known written account of mahjong in any language other than Chinese.
- By 1910, there were written accounts in many languages, including French and Japanese.
- The game was imported to the United States in the 1920s, and became a big success.
- A total of 12,000 mahjong sets were sold quickly.

Babcock's book

- Joseph Park Babcock published his book Rules of Mah-Jongg, also known as the “red book”. This was the earliest version of mahjong known in America.
- Babcock had learned mahjong while living in China.
- Babcock's rules simplified the game to make it easier for Americans to take up, and his version was common through the mahjong fad of the 1920s.
- There were even hit songs related to Mahjong in those days, e.g., “Since Ma is Playing Mah Jong” by Eddie Cantor.
- Later, when the 1920s fad died out, many of Babcock's simplifications were abandoned.
- By the 1930s, many revisions of the rules developed that were substantially different from Babcock's classical version.

Mahjong organizations in America

- The National Mah Jongg League (NMJL), founded by Jewish players, in US in 1937.
- It published the first American mahjong rulebook, *Maajh: The American Version of the Ancient Chinese Game*.
- Players usually use the American game as a family-friendly social activity, not as gambling.
- In 1986, the National Mah Jongg League conducted their first Mah Jongg Cruise Tournament, in conjunction with Mah Jongg Madness.
- In 2010, this large scale seagoing event hosts its 25th Silver Anniversary Cruise, with players from all over the States and Canada participating.
- Recently, a second organization has formed, the American Mah Jongg Association. The AMJA currently hosts tournaments all across North America.

Current development

- Today, the popularity and the characteristics of players of mahjong vary from country to country.
- There are also many governing bodies, which often host exhibition games and tournaments.
- It remains far more popular in Asia than in the West.
- Mahjong, as of 2010, is the most popular table game in Japan. There are several manga and anime (e.g. Saki and Akagi) devoted to dramatic and comic situations involving mahjong.
<http://japanese-mahjong.com/mahjong-manga.html>
- Mahjong culture is still deeply ingrained in the Chinese community. People have used mahjong as the themes of pop music and movies in Hong Kong. <http://mahjongmovies.net/wordpress/>
- Studies by doctors have also shown in Hong Kong that the game is beneficial for individuals suffering from dementia or cognitive memory difficulties, leading to the development of mahjong therapy.

The game set

The game is played with a set of 136 tiles:



Circles or Dots numbered 1 to 9.



Characters numbered 1 to 9.



Bamboos numbered 1 to 9.



The East, South, West and North

The Red, Green and White Dragons

Each has 4 copies summing up to $34 * 4 = 136$ pieces.

How to win

- The **objective** is to get a winning hand (with 14 tiles) consisting of 4 melds (3 tildes each) and a pair.

A meld either consists of 3 identical pieces (honors or suits), or 3 consecutive bamboos, 3 consecutive dots, or 3 consecutive character pieces.

- Examples of winning hands.



How to play

- The tiles are shuffled and stacked up to 4 walls in front of the four players.
Each wall has 17 tiles in the lower level and 17 tiles on the top.
- Each player will pick 13 tiles initially.
- Each player takes a turn picking up a tile from the wall and then discarding a tile by throwing it into the centre of the table.
- Play continues this way until one player has a winning hand by picking up the winning piece from the wall, or seeing someone discard a piece that he/she can use to form the winning hand.

Interesting winning patterns

- A special hand with 9 possible winning possibilities.



- A non-trivial example:



- How about a hand with m winning possibilities for $m \in \{1, \dots, 9\}$?

- Construct a hand of “characters” different from the example in class that can win with 7 possible pieces.
- Show that you can or you cannot construct a hand of “characters” that can win with 8 possible pieces. Explain your answer.
- **Extra credit/open problems**
 - 1 Construct a hand of “characters” that can win with m possible pieces with $m \in \{1, \dots, 9\}$.
 - 2 Generate all character hands with m winning pieces for some/all m .
 - 3 Write a computer program which recognizes/generate winning patterns.

Homework solutions of students

1. Here are several hands with 7 possible winning pieces.

a) 1,1,1,2,3,4,5,6,6,6,7,7,7 [fails to win with 8,9]

b) 9,9,9,8,7,6,5,4,4,4,3,3,3 [fails to win with 1,2]

c) 1,2,3,4,5,6,7,8,8,8,9,9,9 [fails to win with 2,5]

d) 9,8,7,6,5,4,3,2,2,2,1,1,1 [fails to win with 8,5]

e) 2,3,4,4,4,4,5,6,6,6,6,7,8 [fails to win with 4,6]

f) 3,3,3,4,4,5,5,6,6,7,8,8,8 [fails to win with 1,5]

g) 7,7,7,6,6,5,5,4,4,3,2,2,2 [fails to win with 9,5]

Can we construct a hand that can win with any pieces but i, j for any $i < j$?

2. Here are several hands with 8 winning pieces:

- a) 2,2,2,3,4,5,6,7,7,7,8,8,8 [fails to win with 9]
- b) 8,8,8,7,6,5,4,3,3,3,2,2,2 [fails to win with 1]
- c) 2,3,4,4,4,4,5,6,7,8,9,9,9 [fails to win with 4]
- d) 8,7,6,6,6,6,5,4,3,2,1,1,1 [fails to win with 6]

Can we construct a hand that can win with any pieces but i ?

3. One can construct hands that can win with only i pieces for $i = 1, \dots, 9$.

- a) 1,1,1,2,2,2,3,3,3,4,4,4,9 [can only win with 9]
- b) 1,1,1,2,2,2,3,3,3,4,4,7,8 [can only win with 6,9]
- c) 1,1,1,4,4,5,5,5,6,6,9,9,9 [can only win with 4,5,6]
- d) 1,1,1,4,4,4,5,5,6,6,9,9,9 [can only win with 4,5,6,7]
- e) 1,1,1,4,4,4,5,6,6,6,9,9,9 [can only win with 3,4,5,6,7]
- f) 1,1,1,3,3,3,4,4,4,5,5,5,6 [can only win with 2,3,4,5,6,7]
- g) 1,1,1,2,3,4,5,6,6,6,7,7,7 [only fails to win with 8,9]
- h) 2,2,2,3,4,5,6,7,7,7,8,8,8 [only fails to win with 9]
- i) 1,1,1,2,3,4,5,6,7,8,9,9,9 [win with any piece]